DIFFERENTIAL STIFFNESS EFFECTS

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Differential stiffness as developed in NASTRAN is a linear change in stiffness caused by applied loads. Examples of differential stiffness are the stiffening effects of gravity forces in a pendulum, centrifugal forces in rotor blades, and pressure loading of shell structures. In cases wherein this stiffness caused by a load is destabilizing, the differential stiffness concept lends itself to nonlinear structural analysis. Rigid Formats 4 (static analysis with differential stiffness) and 13 (normal modes with differential stiffness) are specifically designed to account for such stiffness changes.

This paper clarifies how pressure loading may be treated in these rigid formats. This clarification resulted from modal correlation of ground vibration test (GVT) results for the empty and pressurized filament wound case (FWC) quarter-scale Space Shuttle solid rocket booster (QSSRB). A sketch of the QSSRB cantilevered to the floor by its external tank attachments is shown in Figure 1.

Correlation of GVT modal data with math model predictions for the FWC QSSRB showed frequency errors of 30 percent and 60 percent for fundamental pitch-roll and Z-bending modes (Table I).

To further isolate this discrepancy, a typical ring section of bars was extracted from the QSSRB model. Figure 2 is a sketch of this ring showing 2 of the 16 radial forces applied, representing discretized pressure forces obtained with the FORCE card.

Eigenvalues of this ring for unrestrained in-plane motion showed two zero frequencies for translation modes, and a rigid body rotational mode at a frequency that, instead of being zero, was nearly as high as the first elastic mode frequency. Examination of ring stiffness coefficients in cylindrical coordinates showed that although radial and rotational displacement coefficients were in balance, the tangential displacement coefficients had a moment unbalance equal to the radial force vector times the tangential displacement (for small displacements). Further, this restoring moment, converted to overall ring rotational stiffness, produced a rotational mode frequency matching the third eigenvalue.

Mathematical development of the differential stiffness concept is presented in the *NASTRAN Theoretical Manual*. Stated therein, externally applied loads, included in the computation of differential stiffness coefficients, are assumed to remain constant in magnitude and direction (CMD assumed). The limiting nature of this assumption is not always easily recognized. With hindsight, this rotational mode restoring moment is easy to predict by the CMD assumption, and is illustrated in Figure 3.

Clearly, if differential stiffness effects caused by pressure are to be modeled, representation by loading other than external forces are necessary. One such possible approach is the use of low stiffness rods with high initial strains, i.e., the rod stiffness should be low enough that overall structural characteristics remain unchanged. As a quantitative check, eigenvalues using Rigid Format 3 (normal modes without differential stiffness effects) for the structural configuration with and without the rods could be compared. The initial strains should be high enough that response displacements do not cause rod elastic forces to appreciably change the pressure preloads. Either DEFORM or TEMP cards could create these initial strains.

With some DMAP, a second possibility would be to store the computed differential stiffness matrix for recombination with nominal structure (without pressure rods) in a succeeding step. In the second approach, requirements for low stiffness and high strain would not exist.

Pressure rods used in a single cross section of the QSSRB and an overall view of all the rods are shown in Figure 4. An initial deformation of 50 radii was used for the radial rods. Contiguous rods along the body centerline provided the longitudinal pressure force. The common centerpoint of radial rods at each pressurized cross section also served as a joint in these longitudinal rods. Their total initial deformation was also 50 radii.

Frequencies obtained with these pressure rods are compared in Table II with those of Table I.

Based upon these results, PLOAD and FORCE cards used to apply pressure forces in Rigid Formats 4 and 13 will result in fictitious moment restraints. To circumvent this restriction, pressure forces may be regarded as a self-equilibrating system in the same way as forces arising from initial preload (DEFORM or TEMP card). For pressurized structures of arbitrary shape, the task of defining pressure rods along selected surface normals may not be easy. As a possible aid, MPC might be used to define a rigid platform of suitable shape from which pressure rods can be supported. Of course, this platform will remain in equilibrium if rod forces have been correctly applied. Further, depending upon magnitude of structural displacements, the condition of rod forces remaining normal to the surface may have to be addressed. The analysis would then be iterative.

Short of possible stiffness formulations at the element level, an automated treatment of pressure loading is needed to support the PLOAD and FORCE cards. Much of the needed capability, such as discretization of pressure forces, already exists. The automated definition of corresponding pressure rods, their properties, and initial strains is needed.

TABLE I.—INITIAL CANTILEVERED FWC QSSRB (EMPTY) FREQUENCIES (HZ) AT 500 PSI

Test	Pressure With PLOAD Card	Description
15.13	19.21	Pitch/roll mode
18.80	21.22	Y-bending
22.46	35.26	Z-bending
36.97	38.24	XY
49.61	50.04	
54.10	56.04	

TABLE II.—FINAL CANTILEVERED FWC QSSRB (EMPTY) FREQUENCIES (HZ) AT 500 PSI

Test	Pressure With PLOAD Card	Pressure With DEFORM Card	Description
15.13	19.21	14.94	Pitch/roll mode
18.80	21.22	18.53	Y-bending
22.46	35.26	22.16	Z-bending
36.97	38.24	37.55	XY
49.61	50.04	49.30	
54.10	56.04	53.90	

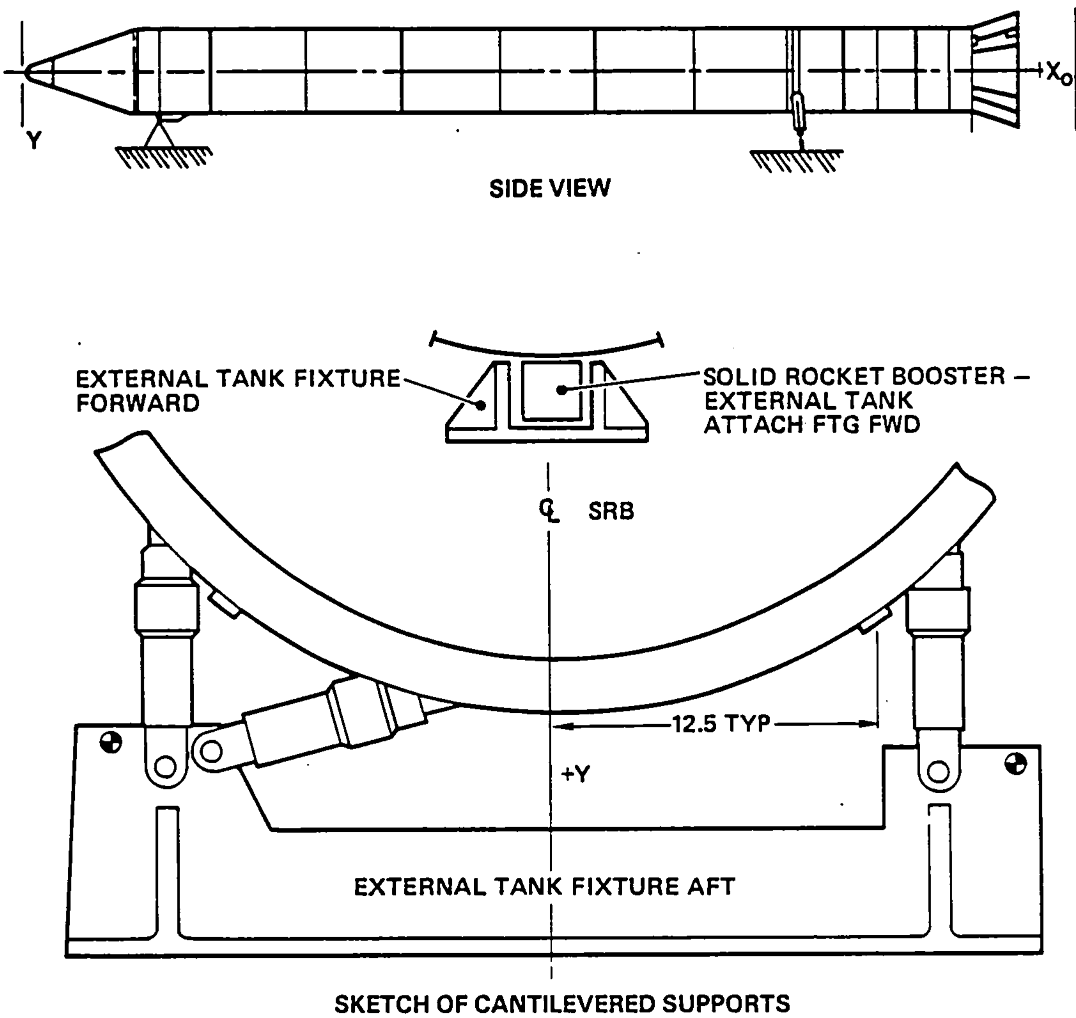


Fig. 1 QSSRB Cantilevered at External Tank Attachments

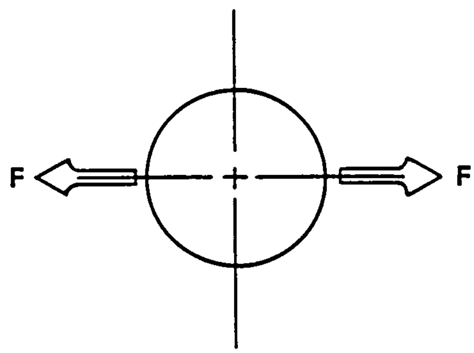
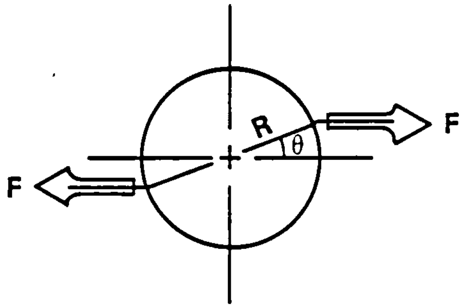
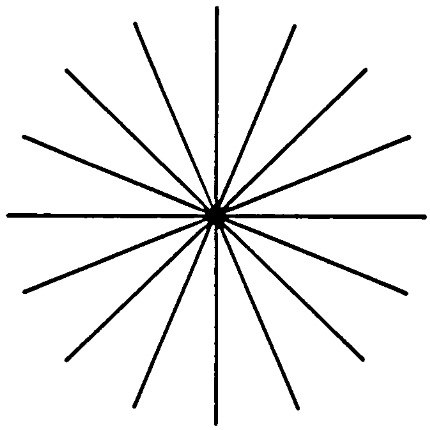


Fig. 2 Sketch of Pressurized Ring Showing 2 of 16 Discretized Pressure Forces

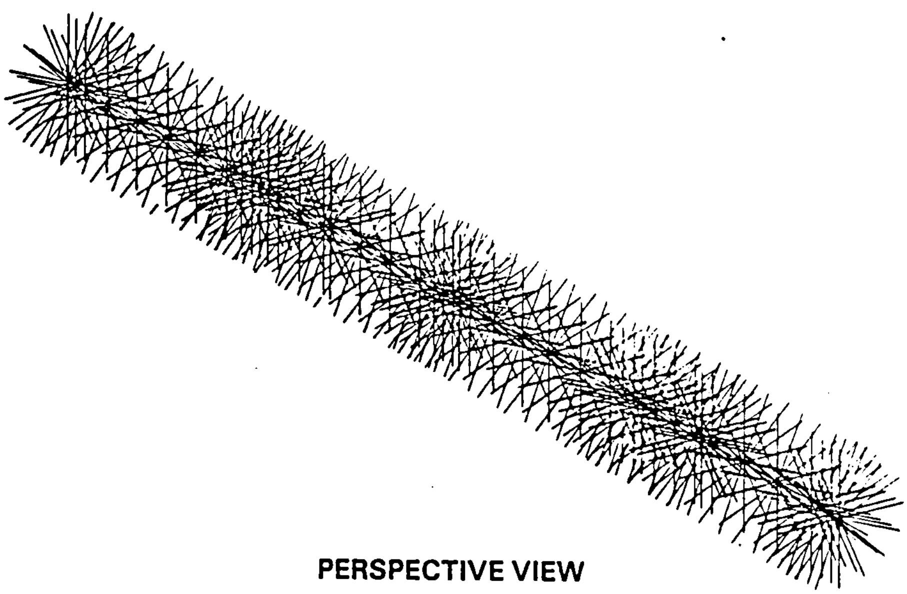


RESTORING MOMENT = 2RFΘ

Fig. 3 Sketch of Pressurized Ring Showing Forces After Rotational Displacement



END VIEW



PERSPECTIVE VIEW

Fig. 4 Pressure Rods in QSSRB Model

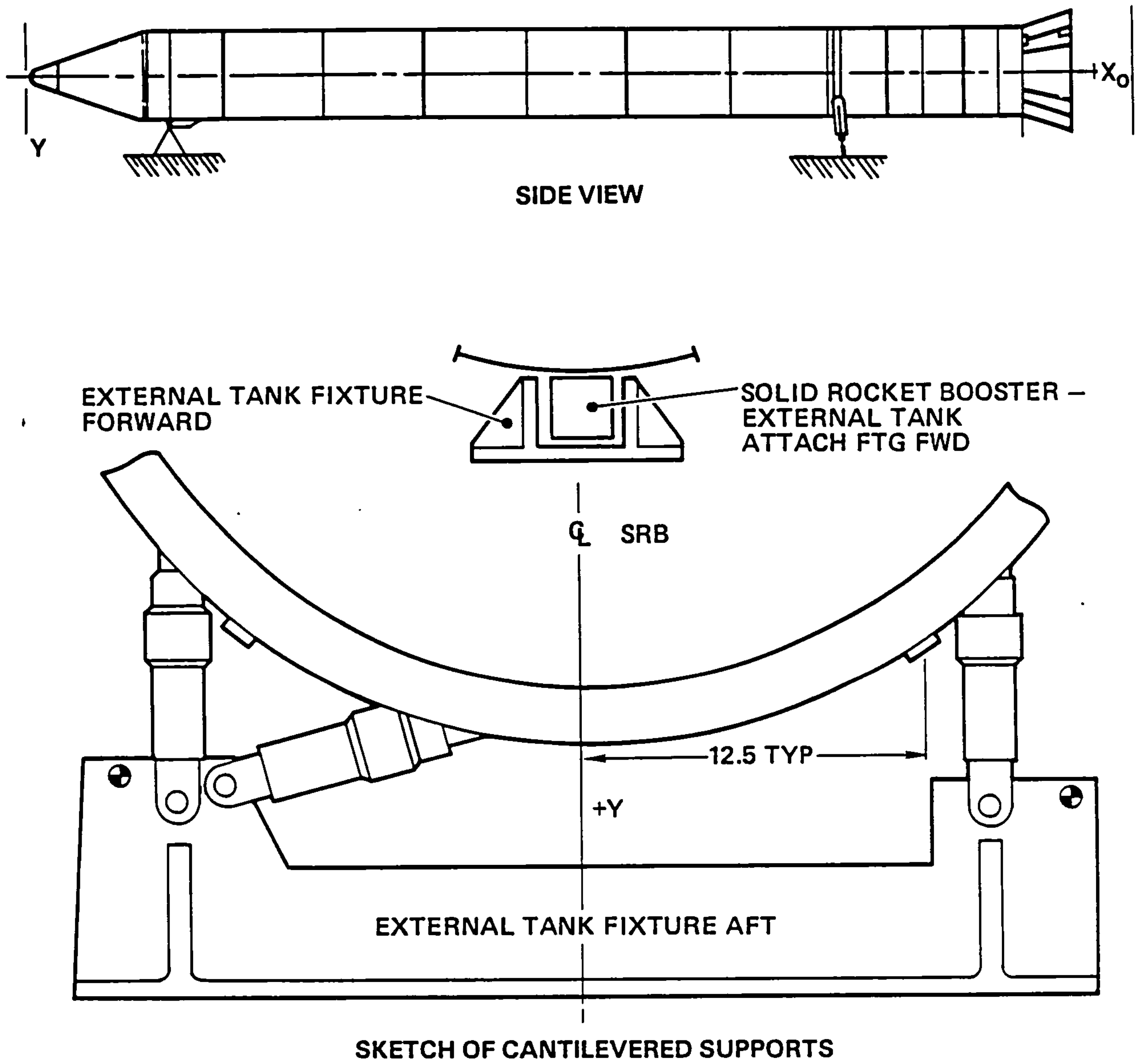


Figure 1. QSSRB Cantilevered at External Tank Attachments

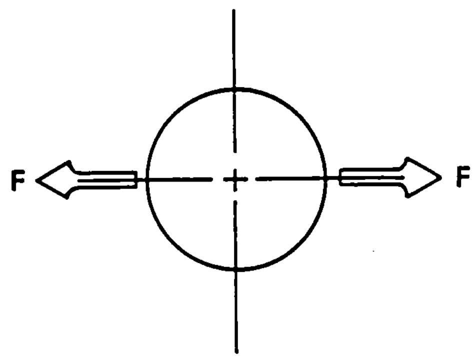
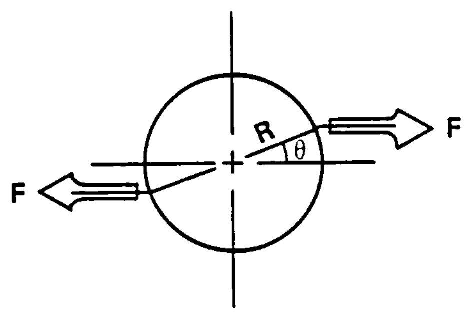


Figure 2. Sketch of Pressurized Ring Showing 2 of 16 Discretized Pressure Forces



RESTORING MOMENT = 2RFΘ

Figure 3. Sketch of Pressurized Ring Showing Forces After Rotational Displacement

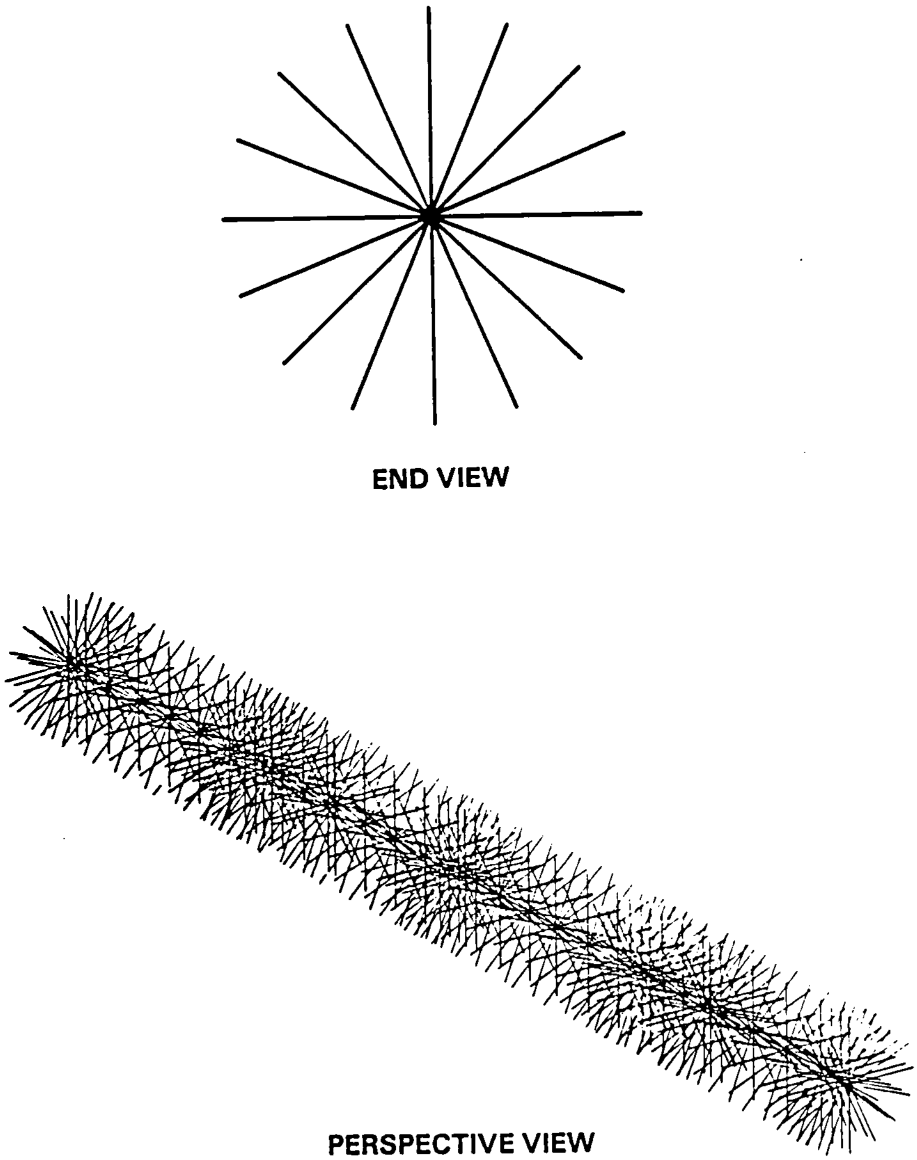


Figure 4. Pressure Rods in QSSRB Model